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$$\frac{du}{dC_1} = \frac{C}{1+C^2C_1^2}, \quad \frac{du}{dC_2} = -\frac{C}{1+C^2C_2^2},$$

$$\frac{d^2u}{dC_1^2} = -\frac{2C^3C_1}{(1+C^2C_1^2)^2}, \quad \frac{d^2u}{dC_2^2} = \frac{2C^3C_2}{(1+C^2C_2^2)^2}, \quad \frac{d^2u}{dC_1dC_2} = 0.$$

$$\text{Now } \frac{d^2u}{dC_1^2} \frac{d^2u}{dC_2^2} < \frac{d^2u}{dC_1dC_2}.$$

Hence there is no maximum unless  $C_1 = \infty$ ,  $C_2 = -\infty$ .

**II. Solution by V. M. SPUNAR, Chicago, Ill., and the PROPOSER.**

By the principle of the instrument, we have

$$\frac{\tan\phi_1}{\tan\phi_2} = \frac{c_1}{c_2}. \quad \therefore \frac{\tan\phi_1 - \tan\phi_2}{\tan\phi_1 + \tan\phi_2} = \frac{c_1 - c_2}{c_1 + c_2} \dots (1).$$

But by trigonometry,

$$\tan\phi_1 - \tan\phi_2 = \frac{\sin(\phi_1 - \phi_2)}{\cos\phi_1 \cos\phi_2}, \quad \tan\phi_1 + \tan\phi_2 = \frac{\sin(\phi_1 + \phi_2)}{\cos\phi_1 \cos\phi_2}.$$

Hence (1) becomes

$$\frac{\sin(\phi_1 - \phi_2)}{\sin(\phi_1 + \phi_2)} = \frac{c_1 - c_2}{c_1 + c_2}, \text{ or } \sin(\phi_1 - \phi_2) = \left(\frac{c_1 - c_2}{c_1 + c_2}\right) \sin(\phi_1 + \phi_2).$$

Hence,  $(\phi_1 - \phi_2)$  is a maximum when  $\sin(\phi_1 + \phi_2)$  is maximum; that is, when  $\sin(\phi_1 + \phi_2) = 1$ ; that is,  $\phi_1 + \phi_2 = \frac{1}{2}\pi$ .

**297. Proposed by PROF. L. C. WALKER, Socorro, New Mexico.**

A square hole 2s on a side is cut through an ellipsoid, axes  $2a$ ,  $2b$ ,  $2c$ , the axis of the hole coinciding with the axis  $2c$  of the ellipsoid. Find (1) the volume, and (2) the surface removed.

**Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

$$\text{The volume } V = 8c \int_0^s dx \int_0^s dy \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2}$$

$$\begin{aligned}
&= \frac{4c}{b} \left[ \frac{bs}{a} \int_0^s \sqrt{\frac{a^2(b^2-s^2)}{b^2} - x^2} + \frac{b^2}{a^2} \int_0^s (a^2-x^2) \sin^{-1} \frac{as}{b} \sqrt{a^2-x^2} dx \right] \\
&= \frac{4bc s}{3a^2} (3a^2 - s^2) \sin^{-1} \frac{as}{b} \sqrt{a^2 - s^2} + \frac{4acs(3b^2 - s^2)}{3b^2} \sin^{-1} \frac{bs}{a} \sqrt{b^2 - s^2} \\
&\quad + \frac{8abcs \sin^{-1} s^2}{3(a^2 - s^2)^{\frac{1}{2}} (b^2 - s^2)^{\frac{1}{2}}} + \frac{10cs^2}{3ab} \sqrt{[a^2b^2 - (a^2 + b^2)s^2]}.
\end{aligned}$$

The surface  $S = \frac{4}{ab} \int_0^s dx \int_0^s dy \sqrt{\frac{a^4b^4 - b^4(a^2 - c^2)x^2 - a^4(b^2 - c^2)y^2}{a^2b^2 - b^2x^2 - a^2y^2}}$ .

The integration of this leads to elliptical functions.

NOTE. If any one will complete the solution of the second part of this problem we shall be pleased to publish it. **Ed. F.**

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### NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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NOTE. Dr. Whalen, of the University of Illinois, has consented to edit this department in the future. Will contributors please send their problems and solutions to him?

174. Proposed by **B. KRAMER**, Student, University of Pittsburgh, Pittsburgh, Pa.

Find a general solution of  $x(x+a)=y^2$ ,  $a$ ,  $x$ , and  $y$  being integers. Given  $a$ , required to find  $x$  to satisfy conditions.

I. Solution by **PROFESSOR F. L. GRIFFIN**, Ph. D., Williams College.

Let  $d$  be any integer contained in  $a$  an odd number of times. Then the general solution is:  $x=(a-d)^2/4d$ ,  $y=(a^2-d^2)/4d$ .

For let  $y=x+n$ , whence  $x^2+ax=x^2+2nx+n^2$ , or  $x=n^2/(a-2n)$ .

Now  $x$  is integral if, and only if,  $a-2n$  is a factor of  $n$ ; that is, if an integer  $k$  exists such that  $n=kd$  [denoting  $a-2n$  by  $d$ ]. But then  $a=d+2n=d(1+2k)$ ; so that integral values of  $x$  exist if, and only if,  $d$  is contained in  $a$  an odd number of times.

Using any such number  $d$  we have,  $k=(a-d)/2d$ ,  $n=\frac{1}{2}(a-d)$ ,  $x=(a-d)^2/4d$ ,  $y=x+\frac{1}{2}(a-d)=(a^2-d^2)/4d$ , as stated.

*Remarks.* (I) Negative values of  $d$  are admissible, as the factors  $x$  and  $x+a$  merely have their numerical values interchanged.

(II) If  $a$  be prime,  $d=\pm 1$  gives the only solution except the trivial case  $d=a$ ,  $x=y=0$ .

(III) Numerical examples follow: